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Green's Function Solution to Radiative Heat Transfer Between Longitudinal Gray Fins

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Nomenclature

b = fin thickness, $=2t$
$f(x_0)$ = minimum dimensional location on adjacent fin where radiation is incident
$f^*(\eta)$ = $f(x_0)/l$
$G(\eta,\eta_0)$ = Green's function; Eq. (6)
k_0 = thermal conductivity at reference temperature T_b
l = total length of fin and tube radius, $=w+R$
N_c = conduction-radiation number, $l^2\sigma T_b^3/(k_0 t)$
$q_{0,x_0}(x_0)$ = dimensional radiosity
$Q_0(\eta)$ = dimensionless radiosity, $Q_0 = q''_{0,x_0}/(\sigma T_b^4)$
R = tube radius
$T(x_0)$ = dimensional temperature
T_b = dimensional tube-base temperature
t = fin half-thickness, $=b/2$
w = fin length
α = angle
β_m = dimensional coefficients required for temperature-dependent thermal conductivity
β_m^* = dimensionless Taylor series coefficients, $\beta_m T_b^m$
$\delta(\eta_0-\eta)$ = Dirac delta function
ϵ = emissivity
γ = opening angle between adjacent fins
η = dimensionless spatial variable, x_0/l
η_0 = dimensionless "dummy" spatial variable
η^* = dimensionless spatial variable, $\eta^*=(\eta-\tau)/(1-\tau)$
$\theta(\eta)$ = dimensionless temperature T/T_b
$\kappa^*(\eta)$ = cutoff angle shown in Fig. 1
λ = dimensionless half-thickness of fin to entire length of radiator, t/l
ξ = dimensionless spatial variable, x_1/l
ρ = reflectivity
σ = Stefan-Boltzmann constant
τ = dimensionless radius to radiator length ratio, R/l

Introduction

OWING to the nonlinear and rather complex nature associated with conductive-radiative heat transfer, the development of new computational tools that can readily and accurately solve this class of problem is salient. The objective of the present study is to demonstrate the applicability and versatility of a pure integral formulation for radiative-conductive heat transfer problems. Preliminary results indicate that this formulation permits an accurate, fast, and stable computational procedure to be implemented. A multidimensional extension of the one-dimensional methodology described within this Note is presently underway.

Several idealized one-dimensional, steady-state fin radiator studies have been expounded upon since the 1960s.[1-5] However, most previous investigations[1-7] have had several shortcomings with regard to their assumptions (no base interactions or solar sources), analysis, and solution. The inclusion of base interactions on a longitudinal fin array was addressed by Schnurr.[7] However, Schnurr presented a limited set of results and did not investigate the effect of a temperature-dependent thermal conductivity. Sparrow and Eckert[2] and Sarabia[5] did include base interactions in the study of tube-sheet radiators.

Analysis

Figure 1 illustrates the physical situation under consideration. The dimensionless tube radius is denoted by τ, and the dimensionless fin length is $(1-\tau)$. A black tube is maintained at the uniform dimensionless temperature of unity, and the ambient temperature is taken as 0 K. The straight, opaque fins are assumed to behave as diffuse gray emitters. Inclusion of a solar source would not affect the analytic-numeric method, though the gray assumption would then become questionable. In this study, a generalized temperature-dependent thermal conductivity[8] has also been included in the analysis. Finally, the fins are assumed to be thin in order to ensure the one-dimensional assumption in the radial direction.

Primitive Formulation

The analytic formulation adopted in this Note appeals to the primitive formulation developed in Refs. 9 and 10, which preserves the direct coupling between the temperature and radiosity. Using the dimensionless quantities shown in the Nomenclature, one can write the dimensionless heat equation as

$$\frac{\mathrm{d}^2\theta(\eta)}{\mathrm{d}\eta^2} = -\sum_{m=1}^{M_{\max}} \frac{\beta_m^*}{(m+1)}\frac{\mathrm{d}^2}{\mathrm{d}\eta^2}[\theta(\eta)-1]^{m+1} + Q_0(\eta)N_c - N_c\int_{\xi=f^*(\eta)}^{1} Q_0(\xi)K_1^*(\eta,\xi)\,\mathrm{d}\xi - N_c\int_{\alpha=0}^{\kappa^*(\eta)} K_2^*(\eta,\alpha)\,\mathrm{d}\alpha \tag{1a}$$

where the first term on the right side of Eq. (1a) accounts for the nonlinear contributions of the temperature-dependent thermal conductivity[8] (note that $\beta_0^*=1$). The shape factors for the surface radiation terms can be expressed as

$$K_1^*(\eta,\xi) = \frac{\sin^2\gamma}{2}\frac{\eta\xi}{(\eta^2+\xi^2-2\eta\xi\cos\gamma)^{3/2}} \tag{1b}$$

$$K_2^*(\eta,\alpha) = \frac{\tau^2\sin\alpha}{2}\left[\frac{\eta\cos\alpha-\tau}{(\tau^2+\eta^2-2\eta\tau\cos\alpha)^{3/2}}\right] \tag{1c}$$

Note that the second integral vanishes as τ goes to 0. This case was considered by Sparrow et al.[1] When this occurs, it is

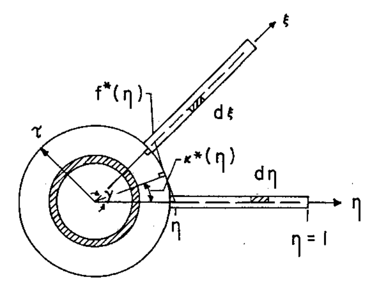

Fig. 1 Dimensionless coordinate system displaying fin configuration.

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evident that the remaining kernal, $K_1^*(\eta,\xi)$, is indeterminant at the origin. Sparrow et al.[1] used physical arguments to obtain the value for the radiosity and irradiation at the origin in order to avoid numerical difficulties. A recent mathematical proof[11] has been developed that confirms the arguments of Sparrow et al.[1] Also, it is clear that, as η approaches τ, $\kappa^*(\eta)\to 0$. This limiting case must be interpreted carefully; that it, the integration must be performed prior to taking the formal limit.

Equation (1a) requires two boundary conditions to be specified. The associated dimensionless boundary conditions can be written as

$$\theta(\eta) = 1, \qquad \eta = \tau \tag{2a}$$

and

$$-\frac{\mathrm{d}\theta(\eta)}{\mathrm{d}\eta} = \sum_{m=1}^{M_{\max}} \frac{\beta_m^*}{(m+1)} \frac{\mathrm{d}}{\mathrm{d}\eta}\left[\theta(\eta)-1\right]^{m+1} + \epsilon\lambda N_c\theta^4(\eta), \qquad \eta = 1 \tag{2b}$$

Finally, the balance of radiant energy can be expressed in dimensionless form as

$$Q_0(\eta) = \epsilon\theta^4(\eta) + (1-\epsilon)\left\{\int_{\xi=f^*(\eta)}^{1} Q_0(\xi)K_1^*(\eta,\xi)\,\mathrm{d}\xi + \int_{\alpha=0}^{\kappa^*(\eta)} K_2^*(\eta,\alpha)\,\mathrm{d}\alpha\right\} \tag{3}$$

It is clear from Eqs. (1a) and (3) that the simultaneous solution of the temperature and radiosity is required. It is also apparent from Eq. (1a) that both differentials and integrals must be discretized in some manner.

Next, it is proposed to transform the nonlinear integro-differential equation shown in Eq. (1a) into a pure nonlinear Fredholm integral equation. The most systematic way to accomplish this transformation is through the use of Green's function method, since Green's function is the natural link between differential and integral forms.[9,10,12]

Accessory Problem Defining Green's Function

The accessory problem (Green's function problem) will allow the immediate transformation of the original expression shown in Eq. (1a) into a pure integral equation for $\theta(\eta)$. Thus, we define the accessory problem as

$$\frac{\mathrm{d}^2}{\mathrm{d}\eta_0^2}\left[G(\eta,\eta_0)\right] = -\delta(\eta_0-\eta), \qquad \eta_0\epsilon(\tau,1) \tag{4a}$$

$$G(\eta,\eta_0) = 0, \qquad \eta_0 = \tau \tag{4b}$$

$$\frac{\mathrm{d}G}{\mathrm{d}\eta_0}(\eta,\eta_0) = 0, \qquad \eta_0 = 1 \tag{4c}$$

where $\partial(\eta_0-\eta)$ is the Dirac delta function.[12]

Integral Equation Representation

Use of the accessory problem as shown in Eqs. (4) permits Eqs. (1) to be transformed into the pure integral equation:

$$\sum_{m=0}^{M_{\max}} \frac{\beta_m^*}{(m+1)}\left[\theta(\eta)-1\right]^{m+1} = -(\eta-\tau)\epsilon\lambda N_c\theta^4(1)I_{\mathrm{end}} - \int_{\eta_0=\tau}^{1} G(\eta,\eta_0)B\left[\eta_0,\theta,(\eta_0),Q_0(\eta_0)\right]\mathrm{d}\eta_0 \tag{5a}$$

where

$$B\left[\eta_0,\theta(\eta_0),Q_0(\eta_0)\right] = N_c\left\{Q_0(\eta_0) - \int_{\xi=f^*(\eta_0)}^{1} Q_0(\xi)K_1^*(\eta_0,\xi)\,\mathrm{d}\xi - \int_{\alpha=0}^{\kappa^*(\eta_0)} K_2^*(\eta_0,\alpha)\,\mathrm{d}\alpha\right\} \tag{5b}$$

and where $I_{\mathrm{end}}=0$ implies that the end is insulated, and $I_{\mathrm{end}}=1$ implies that the tip radiates to free space.

The Green's function, $G(\eta,\eta_0)$, can be resolved by elementary means to produce the two-point function:

$$G(\eta,\eta_0) = \begin{cases}\eta_0-\tau, & \eta_0\epsilon[\tau,\eta] \\ \eta-\tau, & \eta_0\epsilon[\eta,1]\end{cases} \tag{6}$$

Next, a numerical method was implemented for simultaneously solving Eqs. (3) and (5) for the temperature and radiosity distributions along the fin.

Results

The determination of the temperature and radiosity at any discrete location requires the degree of blockage to be resolved. Since some blockage always exists (when $\tau>0$), the extent of the shadowing must be quantified in order to approximate the integrals required in Eqs. (3) and (5). That is, the cutoff angle $\kappa^*(\eta)$ and the first location on an adjacent fin where no blockage occurs, $f^*(\eta)$, must be determined. Once this is accomplished, a quadrature rule can be implemented for approximating the integrals. A general discussion of the numerical procedure has been presented in Ref. 9 and thus will not be restated here. The large number of independent parameters and limited amount of space available dictate that only new results be displayed.

Figure 2 ($\epsilon=0.75$, $M_{\max}=0$)

Results displayed in Fig. 2 indicate the role of tube radius on the establishment of the temperature distribution at two different opening angles. In Fig. 2a, where $\gamma=60$ deg, the fin's temperature increases as the radius of the tube τ increases. This makes sense, since the the hot tube is dominating the fin's field of vision. This figure also indicates that a higher N_c causes a rapid deviation from the idealized zero radius tube. Figure 2b considers a larger opening angle, namely, $\gamma=120$ deg. The temperature distributions produced in this configuration concur with the trends previously observed in Fig. 2a. It is clear that, as γ increases, more radiant emission to the environment is allowed. This trend is accentuated as γ increases until $\gamma=2\cos^{-1}(\tau)$, where no further change will occur.

Figure 3 ($\tau=0.25$, $\epsilon=0.75$, $M_{\max}=1$)

Interpretation of Fig. 3 indicates the role of a temperature-dependent thermal conductivity in establishing the steady-state

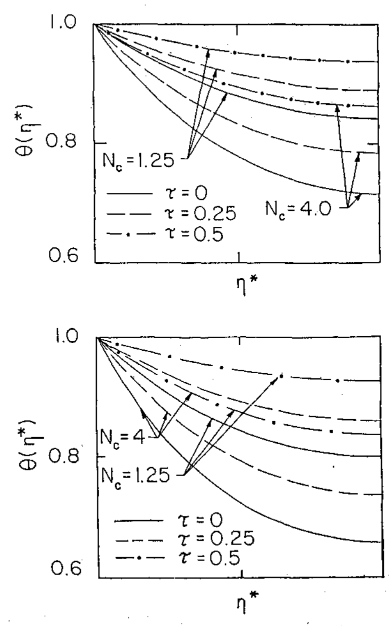

Fig. 2 Dimensionless temperature distribution in a fin when ϵ = 0.75, $M_{\max}$ = 0: a) γ = 60 deg; b) γ = 120 deg.

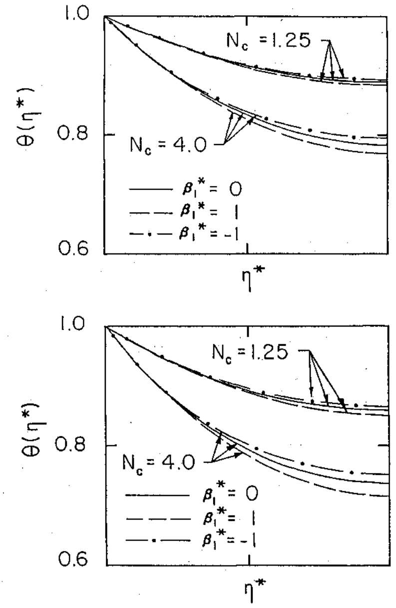

Fig. 3 Dimensionless temperature distribution in a fin when $\tau = 0.25$, $\epsilon = 0.75$, $M_{max} = 1$: a) $\gamma = 60$ deg; b) $\gamma = 120$ deg.

temperature distribution in a fin at two distinct γ. For the sake of clarity, the constant thermal conductivity case has been omitted (this curve would lie within the envelope formed by the $+\beta_1^*$ or $-\beta_1^*$ curves). The thermal behavior depicted by this figure is in line with physical expectations (note the choice of the reference temperature when reviewing this figure). A positive β_1^* tends to lower the temperature, whereas a negative β_1^* has the opposite effect.

Conclusions

The analytic-numeric method used in this Note indicates that complex integro-differential equations of the type that appear in conductive-radiative heat transfer can be recast into a pure integral form that allows an accurate, rapid, and stable computational method to be devised.

Acknowledgment

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Thermal Conductivity Enhancement of Solid-Solid Phase-Change Materials for Thermal Storage

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Introduction

HIGH cell temperatures are seen as the primary safety problem for the Li-BCX battery, which has been used as the electrical supply source for the astronauts' space suit. The exothermic heat from the chemical reactions could raise the temperature of the lithium electrode above lithium's melting point (179°C), resulting in destruction of the battery pack and burn danger to the astronaut. Also, high temperature causes the cell efficiency to decrease.[1] This study was initiated to investigate a thermal storage system in which the batteries would be packed in solid-solid phase-change materials to ensure lower battery cell temperature.

Thermal energy storage in various materials can be categorized by systems which either depend on sensible heat accumulation or materials which undergo a primary (or latent) phase change within the temperature range of operation.[2] Those systems which depend on sensible heat typically require larger quantities of working materials, due to the relatively small heat capacity of these materials compared with the latent heat associated with phase transformation. This larger heat capacity generally results in a mass reduction that can be especially important in space-related applications.

Phase-change materials for thermal energy storage are typically salt hydrates or paraffins that absorb large amounts of heat as they melt. Certain molecular crystals undergo solid-state crystal transformations that absorb heat sufficient enough that they may also be used for practical heat storage applications. The advantages of solid-solid phase transformation are that a liquid phase need not be contained, segregation of components is less likely, and stable composites may be fabricated in which the solid-solid phase-change material is dispersed.

Pentaglycerine ($C_5H_{12}O_3$), PG, and neopentyl glycol ($C_5H_{12}O_2$), NPG, exhibit crystalline transformations that reversibly absorb large amounts of energy.[3] These compounds are of potential use in thermal energy storage components and

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